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THE LAW OF MASS AND ENERGY TRANSFER

P. K. Konakov

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The author proposes a generalization of the known particular laws of mass and energy transfer and their extension to non-equilibrium processes.

In recent years thermodynamic methods have been widely used to investigate mass and energy transfer processes. This has resulted in the development of a branch of science, called the thermodynamics of irreversible processes, whose main hypothesis, local thermodynamic equilibrium, enables the first and second laws of thermodynamics to be applied to nonequilibrium systems.

The new principles in the thermodynamics of irreversible processes are the linear law and the Onsager reciprocity relation.

According to the linear law, the flux I_i due to forces X_i is proportional to these forces:

$$I_i = \sum_{k=1}^{k=n} L_{ik} X_k \quad (i = 1,\ 2,\ \ldots,\ n).$$

The linear law has no theoretical basis, being purely empirical.

The Onsager reciprocity relation is formulated as

$$L_{ik} = L_{ki} \quad (i,\ k = 1,\ 2,\ \ldots,\ n).$$

The relation written indicates that the separate irreversible processes are interrelated. They are superposed one on another and give additive mass and energy transfer effects.

In [1] the new principles of thermodynamics of irreversible processes were applied to the separation of molecular solutions and gaseous mixtures, the authors managing without the second law of thermodynamics.

In [2] the thermodynamics of irreversible processes was applied to flow of a monocomponent liquid through a porous medium. Here the linear law was used as an independent principle unrelated to the second law of thermodynamics.

In the thermodynamics of irreversible processes, the second law is formulated essentially in the same way as in classical thermodynamics. For the thermodynamics of irreversible processes this formulation is insufficient, and we are compelled to introduce the linear law as a new independent principle.

The introduction of the linear law without relation to the second law is a definite shortcoming of the thermodynamics of irreversible processes.

Even in a more specific formulation [3], the second law remains a particular law that determines only temperature heat transfer processes.

For the theory of irreversible processes, a mass and energy transfer law is required, the generalization of which would correspond to the laws of conservation of mass and energy.

The author has attempted [4, 5, 3] to formulate the law of mass and energy transfer in a more general form than usual. In the papers cited assumptions were made regarding the mass and energy capacity of a physical body. These assumptions are essentially identical and may be combined into one assumption about the spatial capacity of the physical body.

Increase of the spatial capacity of the physical bodies participating in mass and energy transfer processes leads to corresponding expressions for the mass and energy transfer vectors. These expressions, however, prove to be only particular expressions, which are valid for steady mass and energy transfer processes, whereas, strictly speaking, they do not hold for unsteady processes.

A more detailed examination of mass and energy transfer processes leads to the conclusion that these processes are determined by variation of the quantity k, which is measured as the product of the body volume V and the time of its existence τ ($k = V\tau$).

The quantity k characterizes the space-time region of existence of the body, and may be called its stereochrone (the space-time continuum of the body).

Let a continuous medium be in an unsteady equilibrium process of change of state. In this medium at the final instant of time τ we select a volume V. The mass M and energy E of the medium in volume V will depend on the values of V and τ.

The process under examination is characterized by the mass and energy transfer of the medium in volume V, to and from the external medium surrounding it. We shall designate by M_S and E_S the amount of mass and energy which pass through the surface S of volume V in time τ, and call them the mass and energy surface fluxes. Clearly, M_S and E_S will be functions of V and τ.

The medium may have mass and energy sources. Let M_V and E_V be the mass and energy liberated in volume V in time τ. The quantities M_V and E_V, called mass and energy source fluxes, will be functions of V and τ.

The equations of conservation of mass and energy for an unsteady equilibrium process of change of state of the medium may be written as

$$M + M_S + M_V = 0, \tag{1}$$

$$\dot{E} + E_S + E_V = 0. \tag{2}$$

The medium included in region V may be in an isochoric, isochronic, or total process of change of state.

In an isochronic process the time of existence of the medium τ remains unchanged, while the volume V of the medium varies.

In an isochoric process the volume V of the medium remains constant, but the time τ of existence of the medium varies.

An isochronic and an isochoric process form the total process of change of state of the medium.

Carrying out partial differentiation of (1) with respect to V, we obtain

$$\frac{\partial M}{\partial V} + \frac{\partial M_S}{\partial V} + \frac{\partial M_V}{\partial V} = 0.$$

The derivatives $\partial M/\partial V$ and $\partial M_V/\partial V$ are, respectively, the density ρ of the medium, and the mass source flux density ρ_V.

The derivative $\partial M_s/\partial V$ is the divergence of the total mass transfer vector $\mathbf{Q}_M$, which is related to time τ:

$$\partial M_S/\partial V = \operatorname{div} \mathbf{Q}_M.$$

Taking the above into account, we may write

$$\rho + \operatorname{div} \mathbf{Q}_M + \rho_V = 0,$$

or

$$\frac{\rho}{\tau} + \operatorname{div} \frac{\mathbf{Q}_M}{\tau} + \frac{\rho_V}{\tau} = 0. \tag{3}$$

Equation (3) is the equation of conservation of mass in an isochronic process.

Carrying out partial differentiation of (1) with respect to τ, and multiplying the sum obtained by V, we have

$$\frac{1}{V}\frac{\partial M}{\partial \tau} + \frac{1}{V}\frac{\partial M_S}{\partial \tau} + \frac{1}{V}\frac{\partial M_V}{\partial \tau} = 0,$$

or

$$\frac{\partial \rho}{\partial \tau} + \operatorname{div} \mathbf{q}_M + \frac{\partial \rho_V}{\partial \tau} = 0, \tag{4}$$

where

$$\mathbf{q}_M = \partial \mathbf{Q}_M/\partial \tau.$$

Equation (4) will be the equation of conservation of mass in an isochoric process.

Combining (3) and (4), we have

$$\frac{\rho}{\tau}+\frac{\partial\rho}{\tau}+\operatorname{div}\left(\frac{\mathbf{Q}_{\mathrm{M}}}{\tau}+\mathbf{q}_{\mathrm{M}}\right)+\frac{\rho_V}{\tau}+\frac{\partial\rho_V}{\partial\tau}=0,$$

or

$$\rho_k+\operatorname{div}\mathbf{q}_{\mathrm{M}k}+\rho_{Vk}=0. \tag{5}$$

Equation (5) is the equation of conservation of mass in the total process.

Differentiating (1) with respect to k, we have

$$\frac{dM}{dk}+\frac{dM_S}{dk}+\frac{dM_V}{dk}=0.$$

Evidently,

$$\frac{dM}{dk}=\frac{\partial M}{\partial V}\frac{\partial V}{\partial k}+\frac{\partial M}{d\tau}\frac{\partial\tau}{dk}=\frac{\rho}{\tau}+\frac{\partial\rho}{\partial\tau}=\rho_{\mathrm{K}},$$

$$\frac{dM_S}{dk}=\operatorname{div}\left(\frac{\mathbf{Q}_{\mathrm{M}}}{\tau}+\frac{\partial\mathbf{Q}_{\mathrm{M}}}{\partial\tau}\right)=\operatorname{div}\mathbf{q}_{\mathrm{M}k},$$

$$\frac{dM_V}{dk}=\frac{\rho}{\tau}+\frac{\partial\rho_V}{\partial\tau}=\rho_{Vk}.$$

The sum of the right sides of these equations gives (5). Similar calculations may be carried out with the equation of conservation of energy (2), and the following modified form obtained:

$$\frac{\varepsilon}{\tau}+\operatorname{div}\frac{\mathbf{Q}_{\mathrm{e}}}{\tau}+\frac{\varepsilon_V}{\tau}=0, \tag{6}$$

$$\frac{\partial\varepsilon}{\partial\tau}+\operatorname{div}\frac{\partial\mathbf{Q}_{\mathrm{e}}}{\partial\tau}+\frac{\partial\varepsilon_V}{\partial\tau}=0, \tag{7}$$

$$\varepsilon_k+\operatorname{div}\mathbf{q}_{\mathrm{e}k}+\varepsilon_{Vk}=0. \tag{8}$$

Here ε, $\mathbf{Q}_{\mathrm{e}}$ and ε_V are the energy density, the vector sum of energy transfer, and the energy source flux density. The discussion has shown that the total mass transfer vector $\mathbf{q}_{\mathrm{M}k}$ is composed of the mass transfer vector in an isochronic process $\mathbf{Q}_{\mathrm{M}}/\tau$ and the mass transfer vector in an isochoric process $\partial\mathbf{Q}_{\mathrm{M}}/\partial\tau$, i.e.,

$$\mathbf{q}_{\mathrm{M}k}=\frac{\mathbf{Q}_{\mathrm{M}}}{\tau}+\frac{\partial\mathbf{Q}_{\mathrm{M}}}{\partial\tau}.$$

The same may be said of the total energy transfer vector

$$\mathbf{q}_{\mathrm{e}k}=\frac{\mathbf{Q}_{\mathrm{e}}}{\tau}+\frac{\partial\mathbf{Q}_{\mathrm{e}}}{\partial\tau}.$$

The above equations were obtained for an unsteady equilibrium process. Using the hypothesis of local equilibrium, it is not hard to show that these equations will also be valid for an unsteady nonequilibrium process.

A viscous physical body changes its mass and energy in space and time.

The space and time changes of mass and energy of the body are related to changes of volume and time of existence of the body, and therefore, to its stereochrones.

Changes of mass and energy of a body require corresponding changes of mass and energy of other bodies surrounding it, producing mass and energy transfer between the bodies.

Mass and energy transfer between the bodies is determined by the law of conservation and transformation of mass and energy, mentioned above.

This law, however, proves insufficient for a complete description of mass and energy transfer processes. The law of mass and energy transfer is also necessary, a generalization of which would correspond to the generalization of the laws of conservation and transformation of mass and energy.

The stereochrones of bodies vary in the process of mass and energy transfer between bodies.

The law of variation of stereochrones of interacting bodies will also be a law of mass and energy transfer.

This law is based on experimental data, generalized in the form of known particular laws of mass and energy transfer.

The law of mass and energy transfer is phrased for equilibrium processes with constant rate of change of the body parameters. The hypothesis of local equilibrium gives a basis for extending it to nonequilibrium processes.

For isochronic, isochoric, or total processes, the law of mass and energy transfer may be expressed thus: the total change of the stereochrone of physical bodies taking part in an isochronic, isochoric or total mass and energy transfer process has the largest positive value.

Let us take n physical bodies participating in a mass and energy transfer process.

Let the stereochrone of the i-th body have the value k_i, and receive the increment Δk_i in the mass and energy transfer process.

The law of mass and energy transfer requires that the relation

$$\sum_{i=1}^{i=n} \Delta k_i = + \max \tag{9}$$

hold in the processes concerned.

The right side of the equation represents the greatest positive value under the conditions of the process.

We shall designate by $\varkappa_i$ the specific increment of the stereochrone per unit volume of the i-th body per unit time. Clearly,

$$\varkappa_i = \Delta\varkappa_i/\varkappa_i.$$

On the basis of the law of mass and energy transfer we may write

$$\sum_{i=1}^{i=n} \varkappa_i = + \max, \tag{10}$$

i.e., the total specific change of the stereochrone of the physical bodies taking part in an isochronic, isochoric or total mass and energy transfer process has the greatest positive value.

We shall apply the law of mass and energy transfer to a continuous medium. The unsteady nonequilibrium process of change of state of a continuous medium may be represented in the form of a spatial series of processes in unsteady equilibrium in space, each proceeding in a local volume v.

An unsteady process in equilibrium in space in a local volume v may be represented in the form of a time series of unsteady processes in equilibrium in time, each existing in volume v in a local time interval τ_V with severally constant rates of change of parameters of state of the medium.

The process of change of a constant medium in a local volume v during a local time interval τ_V is a local process.

The total process of change of state of a medium, as has been pointed out above, comprises an isochoric and an isochronic process.

In an isochronic process of change of state of a continuous medium, the stereochrone changes because of an increment in the volume of the medium its transport by the moving medium, and also because of the action of the mass sources of the medium.

In the process described the volume V of the medium receives an increment v, with constant time of existence τ, in consequence of which the mass content M of the medium per unit volume changes by an amount $\partial M/\partial V$, equal to ρ.

The mass increment per unit stereochrone is determined by the derivative $\left(\frac{\partial M}{\partial k}\right)_\tau$. Since

$$\left(\frac{\partial M}{\partial k}\right)_\tau = \frac{\partial M}{\partial V}\left(\frac{\partial v}{\partial k}\right)_\tau$$

and

$$k = V\tau,$$

$$\left(\frac{\partial M}{\partial k}\right)_\tau = \frac{\rho}{\tau}.$$

Consequently, a volume increment V, equal to unity, causes stereochrone increment equal to $\rho/(\rho/\tau)$.

The mass transfer process in an isochronic process is characterized by the total transfer vector $\mathbf{Q}_M$.

We shall divide this vector by ρ/τ. The result is a vector $\mathbf{Q}_M/(\rho/\tau)$, whose modulus is equal to the amount of the stereochrone carried by the medium through unit area of an area element located perpendicular to the direction of motion of the medium, during the time of existence of the medium τ. The vector $\mathbf{Q}_M/(\rho/\tau)$ may be called the total stereochrone transfer vector.

The medium sources per unit volume give a medium mass ρ_V, which causes a change $\rho_V/(\rho/\tau)$ in the medium stereochrone.

In an isochoric process of change of state of a continuous medium, the stereochrone changes due to an increment of time of existence of the medium, its transport by the moving medium, and due to the action of the medium's mass and energy sources.

The increment in medium existence time changes the medium mass per unit volume v per unit time by an amount $\partial_\rho/\partial\tau$.

The mass increment per unit stereochrone per unit volume v is determined by the derivative $\left[\frac{\partial(\rho v)}{\partial k}\right]_V$.

Since

$$\left[\frac{\partial(\rho v)}{\partial k}\right]_V = \frac{\partial(\rho v)}{\partial\tau}\left(\frac{\partial\tau}{\partial k}\right)_V$$

and

$$k = v\tau,$$

$$\left[\frac{\partial(\rho v)}{\partial k}\right]_V = \frac{\partial\rho}{\partial\tau} = \vartheta_\rho.$$

Therefore, a time increment τ, equal to unity, causes a stereochrone increment equal to $\frac{\partial\rho}{\partial\tau}\Big/\vartheta_\rho$.

The mass transfer process in an isochoric process is characterized by the transfer vector $\mathbf{q}_M$. We shall divide this vector by the rate of change of mass density ϑ_ρ.

The result is the vector $\mathbf{q}_M/\vartheta_\rho$, whose modulus is equal to the amount of stereochrone carried by the medium through unit area of an area element situated normal to the direction of motion of the medium, per unit time. The vector $\mathbf{q}_M/\vartheta_\rho$ may be called the stereochrone transfer vector.

The action of the medium sources changes the stereochrone of the medium by the amount $(\partial\rho_V/\partial\tau)/\vartheta_\rho$ per unit volume and unit time.

In a total change of state process, a change of stereochrone arises from a change of the volume of the medium and the time of its existence, from transport of the stereochrone by the moving medium, and from a change of stereochrone due to the action of mass and energy sources.

The change of medium volume and of its existence time change the stereochrone of the medium by an amount ρ_{Vk}/ρ_k.

The total stereochrone transfer vector is

$$\mathbf{q}_{Mk}/\rho_k.$$

Mass sources cause a stereochrone change of ρ_{Vk}/ρ_k.

Let us turn to an isochrone change of state process, in which the existence time τ of the medium remains unchanged, but the volume V changes by an amount v.

An increment of volume V equal to v causes an increment of the stereochrone of the medium equal to $\rho v/(\rho/\tau)$.

A local volume surface increment $d\sigma$ causes an increment of mass surface flux $Q_{M.S}\, d\sigma$, which changes the stereochrone of the medium surrounding volume v by an amount $Q_{M.S}\, d\sigma/(\rho/\tau)$.

For the whole surface σ the change is $\int\limits_{\sigma} \frac{Q_{M.S}}{\rho/\tau}\, d\sigma$.

The change of the stereochrone of the medium surrounding volume v due to the action of mass sources has the value $\rho_V v/(\rho/\tau)$.

On the basis of the law mentioned above, it may be asserted that for the total change of stereochrone of the medium surrounding local volume v, and of the medium in volume v, we have the relation

$$\Delta_V k = \frac{\rho v\tau}{\rho} + \tau \int\limits_{\sigma} \frac{Q_{M.S}}{\rho} d\sigma + \frac{\rho_V v\tau}{\rho} = +\max.$$

Hence

$$\varkappa_V = \frac{\rho}{\rho} + \operatorname{div}\left(\frac{\mathbf{Q}_M}{\rho}\right) + \frac{\rho_V}{\rho} = +\max.$$

On the right side of these relations appears the largest positive value in the given conditions of the process.

Since

$$\operatorname{div}\left(\frac{\mathbf{Q}_M}{\rho}\right) = \frac{1}{\rho}\operatorname{div}\mathbf{Q}_M + \left(\mathbf{Q}_M,\ \operatorname{grad}\frac{1}{\rho}\right),$$

$$\varkappa_V = \frac{1}{\rho}(\rho + \operatorname{div}\mathbf{Q}_M + \rho_V) + \left(\mathbf{Q}_M,\ \operatorname{grad}\frac{1}{\rho}\right) = +\max,$$

or, taking (3) into account, we may write

$$\varkappa_V = \left(\mathbf{Q}_M,\ \operatorname{grad}\frac{1}{\rho}\right) = +\max.$$

Evidently,

$$\operatorname{grad}\frac{1}{\rho} = -\frac{1}{\rho^2}\operatorname{grad}\rho,$$

which gives

$$\varkappa_V = -\frac{1}{\rho^2}(\mathbf{Q}_M,\ \operatorname{grad}\rho) = +\max. \tag{11}$$

Hence

$$(\mathbf{Q}_M,\ \operatorname{grad}\rho) = -\max,$$

i.e., the scalar product of vectors $\mathbf{Q}_M$ and grad ρ has the largest negative value.

The relation obtained requires that vector $\mathbf{Q}_M$ be parallel to the vector grad ρ and oppositely directed, i.e., to ensure the equality

$$\mathbf{Q}_M = -A_M \operatorname{grad}\rho. \tag{12}$$

Equality (12) also determines the absolute value and the direction of the total mass transfer vector in a continuous medium in an isochronic process.

In the equality stated, the variable A_m characterizes the mass transfer intensity in an isochronic process and may therefore be called the mass transfer coefficient. This coefficient is an increasing function of the existence time τ of the medium.

Dividing both sides of (12) by τ, we obtain

$$\frac{Q_{\text{M}}}{\tau} = -\frac{A_{\text{M}}}{\tau}\,\text{grad}\,\rho. \tag{13}$$

The vector $\mathbf{Q}_{\text{M}}/\tau$ is the specific mass transfer vector in the isochrone process. It relates to the final instant of the existence time τ of the medium.

Above we established relation (11). It determines the value of the specific increment of stereochrone $\varkappa_V$.

Substituting (12) into the left side of (11), we obtain

$$\varkappa_V = A_{\text{M}}(\text{grad}\,\rho/\rho)^2. \tag{14}$$

Relation (14) states that the transfer coefficient A_m is a real positive value.

Let us now examine an isochoric change of state process.

We recall that in this process volume V remains unchanged, while the existence time τ of the medium is changed by the value of the local interval τ_V.

A change of medium mass in volume V in unit time, equal to $\int_V \vartheta_\rho dV$, changes the stereochrone of the medium in unit time by an amount $\int_V \frac{\vartheta_\rho}{\vartheta_\rho} dV$.

Through the surface element dS of volume V at the instant of time τ in unit time, a mass $q_{\text{M.S}}\, dS$ of medium flows. This mass changes the stereochrone of the medium surrounding the volume by an amount $q_{\text{M.S}}\, dS/\vartheta_\rho$ in unit time.

For the whole surface S, the change in question will be

$$\int_S q_{\text{M.S}}\, dS/\vartheta_\rho \quad \text{or} \quad \int_V \text{div}\,(\mathbf{q}_{\text{M}}/\vartheta_\rho)\, dV.$$

The change of stereochrone of the medium surrounding volume V due the medium sources in unit time will be

$$\int_V \frac{m_V}{\vartheta_\rho} dV, \quad m_V = \frac{\partial \rho_V}{\partial \tau}.$$

From the law of mass and energy transfer, it may be stated that for the total change of stereochrone of the medium surrounding region V, and of the medium in region V, in unit time we have the relation

$$V\varkappa_\tau = \int_V \frac{\vartheta_\rho}{\vartheta_\rho} dV + \int_V \text{div}\left(\frac{\mathbf{q}_{\text{M}}}{\vartheta_\rho}\right) dV + \int_V \frac{m_V}{\vartheta_\rho} dV = +\max.$$

On the right side of this relation there is the greatest positive value in the given conditions of motion of the medium. Since this relation is valid for a sufficiently small volume V,

$$\varkappa_\tau = \frac{\vartheta_\rho}{\vartheta_\rho} + \text{div}\left(\frac{\mathbf{q}_{\text{M}}}{\vartheta_\rho}\right) + \frac{m_V}{\vartheta_\rho} = +\max.$$

Recalling the formula

$$\text{div}\left(\frac{\mathbf{q}_{\text{M}}}{\vartheta_\rho}\right) = \frac{1}{\vartheta_\rho}\text{div}\,\mathbf{q}_{\text{M}} + (\mathbf{q}_{\text{M}},\ \text{grad}\,\vartheta_\rho),$$

we obtain

$$\varkappa_\tau = \frac{1}{\vartheta_\rho}(\vartheta_\rho + \text{div}\,\mathbf{q}_{\text{M}} + m_V) + (\mathbf{q}_{\text{M}},\ \text{grad}\,\vartheta_\rho) = +\max,$$

or, taking (6) into account, we obtain

$$\varkappa_\tau = \left(\mathbf{q}_{\text{M}},\ \text{grad}\frac{1}{\vartheta_\rho}\right) = +\max.$$

Evidently,

$$\operatorname{grad}\frac{1}{\vartheta_\rho} = -\frac{1}{\vartheta_\rho^2}\operatorname{grad}\vartheta_\rho.$$

Therefore

$$\varkappa_\tau = -\frac{1}{\vartheta_\rho^2}(\mathbf{q}_{\text{M}}, \operatorname{grad}\vartheta_\rho) = +\max. \tag{15}$$

Hence

$$(\mathbf{q}_{\text{M}}, \operatorname{grad}\vartheta_\rho) = -\max,$$

i.e., the scalar product of vectors $\mathbf{q}_{\text{M}}$ and grad ϑ_ρ must have the largest negative value.

The relation obtained requires that the equality

$$\mathbf{q}_{\text{M}} = -A_{\text{M}}\operatorname{grad}\vartheta_\rho \tag{16}$$

hold.

We note that in (12) and (16) the transfer coefficients A_m are identical, since they are determined by identical values of the existence time τ of the medium.

Let us return to relation (15). This relation gives the value of change of stereochrone of the medium per unit volume and unit time in the isochoric process.

Allowing for (16), we may write (15) in the form

$$\varkappa_\tau = A_{\text{M}}(\operatorname{grad}\vartheta_\rho/\vartheta_\rho)^2. \tag{17}$$

Relation (17) again shows that the transfer coefficient A_m is a real positive value.

The isochronic and isochoric processes form the total process of change of state of the medium.

The specific mass transfer vector in the total process $\mathbf{q}_{\text{M}k}$ is equal to the sum of vectors (13) and (16)

$$q_{\text{M}k} = -\frac{A_{\text{M}}}{\tau}\operatorname{grad}\rho - A_{\text{M}}\operatorname{grad}\vartheta_\rho. \tag{18}$$

This same expression may be obtained from the following considerations.

In the total process a change of mass of medium in volume V, equal to $\int\limits_V \dot{\rho}_k\,dV$, changes the stereochrone of the medium in volume V by an amount $\int\limits_V \frac{\dot{\rho}_k}{\rho_k}\,dV$.

Through the surface element dS of volume V at time τ there flows in unit time a mass of medium $q_{\text{M}k\text{S}}\,dS$, which changes the stereochrone of the medium surrounding volume V by an amount $(q_{\text{M}k\text{S}}/\rho_k)dS$.

For the whole surface S the change in question is

$$\int\limits_V \operatorname{div}(\mathbf{q}_{\text{M}k}/\rho_k)\,dV.$$

The change of stereochrone of the medium surrounding volume V due to the action of sources of the medium in unit time is equal to $\int\limits_V (m_{Vk}/\rho_k)\,dV$.

From the law of mass and energy transfer we may write the relation

$$\int\limits_V \frac{\dot{\rho}_k}{\rho_k}\,dV + \int\limits_V \operatorname{div}\frac{\mathbf{q}_{\text{M}k}}{\rho_k}\,dV + \int\limits_V \frac{m_{Vk}}{\rho_k}\,dV = +\max.$$

This relation is valid for an arbitrary object. Therefore

$$\frac{\dot{\rho}_k}{\rho_k} + \operatorname{div}\frac{\mathbf{q}_{\text{M}k}}{\rho_k} + \frac{m_{Vk}}{\rho_k} = +\max.$$

Taking (5) into account, we have

$$(\mathbf{q}_{Mk},\ \mathrm{grad}\,\rho_k') = -\max.$$

This relation leads to the expression

$$\mathbf{q}_{Mk} = -A_M\,\mathrm{grad}\,\rho_k. \tag{19}$$

which also determines the total mass transfer vector.

Since

$$\rho_k = \frac{\rho}{\tau} + \frac{\partial\rho}{\partial\tau},$$

(19) is identical to (18).

Expression (18) shows that the mass transfer vector is determined not only by the mass density gradient, but also by the gradient of rate of change of mass density.

If the medium is in a steady nonequilibrium change of state process, the equality $\partial\rho/\partial\tau = 0$ holds, and (18) takes the form

$$\mathbf{q}_{Mk} = -\frac{A_M}{\tau}\,\mathrm{grad}\,\rho.$$

For sufficiently large $\tau\,|\tau\to\infty|$, A_m becomes sufficiently large $|A_M \to \infty|$, in consequence of which $A_M/\tau = \infty/\infty$. Removing the indeterminacy, we obtain

$$A_M/\tau = \partial A_M/\partial\tau.$$

In the case examined

$$\mathbf{q}_{Mk} = \frac{\mathbf{Q}_M}{\tau} = \frac{\partial\mathbf{Q}_M}{\partial\tau} = q_M.$$

Therefore

$$q_M = -\frac{\partial A_M}{\partial\tau}\,\mathrm{grad}\,\rho.$$

Introducing the notation $\partial A_M/\partial\tau = a_M$, we have

$$\mathbf{q}_M = -a_M\,\mathrm{grad}\,\rho. \tag{20}$$

This expression also determines the mass transfer vector in an unsteady process that is only slightly different from the steady.

Let the medium be in a steady change of state process, i.e., let the medium be in a state of equilibrium during an infinitely large time interval. In this case the total mass transfer vector equals zero, and from (18) we obtain the equation

$$-\frac{A_M}{\tau}\,\mathrm{grad}\,\rho - A_M\,\mathrm{grad}\,\vartheta_\rho = 0.$$

This equation shows that in a medium in a prolonged equilibrium state, two mass transfer vectors exist; these are equal in modulus but opposite in direction.

Above we have examined mass transfer processes in a continuous medium. A similar examination may also be made for energy transfer processes in a continuous medium. Such an examination leads to the relation

$$q_{ek} = -\frac{A_e}{\tau}\,\mathrm{grad}\,\varepsilon - A_e\,\mathrm{grad}\,\frac{\partial\varepsilon}{\partial\tau}. \tag{21}$$

This relation shows that the total energy transfer vector is determined not only by the energy density gradient, but also by the gradient of rate of change of energy density.

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Institute of Railroad Transport Engineers, Moscow